МИНИСТЕРСТВО ОБРАЗОВАНИЯ РОССИЙСКОЙ ФЕДЕРАЦИИ

МИНИСТЕРСТВО РОССИЙСКОЙ ФЕДЕРАЦИИ ПО АТОМНОЙ ЭНЕРГИИ

МИНИСТЕРСТВО ПРОМЫШЛЕННОСТИ, НАУКИ И ТЕХНОЛОГИЙ
РОССИЙСКОЙ ФЕДЕРАЦИИ

РОССИЙСКАЯ АССОЦИАЦИЯ НЕЙРОИНФОРМАТИКИ

МОСКОВСКИЙ ИНЖЕНЕРНО-ФИЗИЧЕСКИЙ ИНСТИТУТ
(ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ)

НАУЧНАЯ СЕССИЯ МИФИ–2003

НЕЙРОИНФОРМАТИКА–2003

**V ВСЕРОССИЙСКАЯ
НАУЧНО-ТЕХНИЧЕСКАЯ
КОНФЕРЕНЦИЯ**

ЛЕКЦИИ ПО НЕЙРОИНФОРМАТИКЕ

Часть 2

По материалам Школы-семинара
«Современные проблемы нейроинформатики»

Москва 2003

УДК 004.032.26 (06)
ББК 32.818я5
М82

НАУЧНАЯ СЕССИЯ МИФИ–2003. V ВСЕРОССИЙСКАЯ НАУЧНО-ТЕХНИЧЕСКАЯ КОНФЕРЕНЦИЯ «НЕЙРОИНФОРМАТИКА–2003»: ЛЕКЦИИ ПО НЕЙРОИНФОРМАТИКЕ. Часть 2. – М.: МИФИ, 2003. – 180 с.

В книге публикуются тексты лекций, прочитанных на Школе-семинаре «Современные проблемы нейроинформатики», проходившей 29–31 января 2003 года в МИФИ в рамках V Всероссийской конференции «Нейроинформатика–2003».

Материалы лекций связаны с рядом проблем, актуальных для современного этапа развития нейроинформатики, включая ее взаимодействие с другими научно-техническими областями.

Ответственный редактор
Ю. В. Тюменцев, кандидат технических наук

ISBN 5–7262–0471–9

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КЛАССИЧЕСКИЕ И КВАНТОВЫЕ МОДЕЛИ МЫШЛЕНИЯ, ОСНОВАННЫЕ НА P-АДИЧЕСКОМ ПРЕДСТАВЛЕНИИ ИНФОРМАЦИИ

Аннотация

Развивается квантовый формализм (вероятностное исчисление в гильбертовом пространстве) для измерений над когнитивными системами). В частности, этот формализм используется для математического моделирования сознания как само-измеряющей (квантово-подобной) системы. С помощью этого формализма можно предсказывать средние значения когнитивных наблюдаемых. Основываясь на фундаментальном результате нейрофизиологических и психологических исследований о иерархической структуре когнитивных процессов, мы используем p-адические иерархические деревья в качестве математической модели классического ментального пространства. Обсуждается проблема выбора адекватной ментальной геометрии. Использование p-адических чисел позволяет описать топологически (с помощью p-адической ультраметрики) иерархическое дерево, используемое для кодирования когнитивной информации. Важнейшей особенностью p-адической модели мышления является наличие алгебраической структуры на p-адическом дереве. Эта структура дает возможность использовать (неархимедов=ультраметрический) математический анализ для описания мыслительных процессов. В частности, на классическом уровне мы используем уравнения в частных производных на p-адических деревьях для описания потоков мыслей. На квантовом уровне мы используем аналог уравнения Шрёдингера (на дереве мыслей) для описания динамики вероятностных распределений. Важнейшим нейрофизиологическим следствием p-адической модели является представление когнитивной информации с помощью иерархических цепей нейронов.

 УДК 004.032.26 (06) Нейронные сети

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CLASSICAL AND QUANTUM MENTAL MODELS BASED ON P-ADIC REPRESENTATION OF INFORMATION

Abstract

We develop a kind of quantum formalism (Hilbert space probabilistic calculus) for measurements performed over cognitive systems. In particular, this formalism is used for mathematical modeling of functioning of consciousness as a self-measuring quantum-like system. By using this formalism we could predict averages of cognitive observables. Reflecting the basic idea of neurophysiological and psychological studies on a hierarchic structure of cognitive processes, we use p-adic hierarchic trees as a mathematical model of a mental space. We also briefly discuss the general problem of the choice of adequate mental geometry.

1. Introduction

Since the creation of quantum mechanics, there are continuous discussions on possible connections between quantum and mental phenomena. During the last hundred years, there was presented a huge number of various proposals and speculations. We shall mention just a few of them.

The philosophic system of *Whitehead* [1]–[3] was the first attempt to establish quantum–mental (or more precisely *mental → quantum*) connection. In *Whitehead's* philosophy of organism "quantum" was some feature of basic protomental elements of reality, namely *actual occasions,* see [1], especially pp. 401–403. See also *A. Shimony* [4] for modern reconsideration of quantum counterpart of *Whitehead's* philosophy of organism. It is especially important for us to underline that all protomental elements of reality have quantum temporal structure:

> "The actual entity is the enjoyment of a certain quantum of physical time."
> – [1], p. 401.

The extended discussion on quantum–mental connection was induced by attempts to solve the problem of quantum measurements, see e. g. [5]–[12]. The most extreme point of view is that physical reality is, in fact, created by acts of observations. This kind of considerations is especially closely related to so called orthodox Copenhagen interpretation of quantum mechanics [1]. By this interpretation a wave function provides the complete description of an *individual* quantum system. An act of measurement induces collapse of the wave function. The problem of measurement is still unsolved in quantum framework (at least on the basis of the conventional interpretation of quantum mechanics, see also section 10). Among various attempts to provide a reasonable explanation of wave function reduction, there should be mentioned attempts to use consciousness as the determining factor of reductions of wave functions, see e. g. *Wigner* [8].

There were also various attempts reduce an act of thinking to quantum collapse, see e. g. *Orlov* [13] (quantum logic of consciousness); see also *Penrose* [14], [15]:

> "I am speculating that the action of conscious thinking is very much tied up with the resolving out of alternatives that were previously in linear superposition."

In fact, *Penrose* worked in the reductionist approach, see e. g. [16] (and compare e. g. [17]–[20]): It seems we could not reduce cognitive phenomena to physical activity of neurons. It might be that we could reduce it to activity of quantum systems. Roughly speaking an act of thinking is reduced to the collapse of wave function in quantum gravity. Our thinking ability is based on collapses of superpositions of two mass states.

The idea of quantum-physical reduction for cognitive processes is quite popular in quantum community. We also mention the investigations of *H. Stapp* [21] who used Copenhagen (Heisenberg-potentiality) approach to quantum mechanics. He also use quantum reductionist approach:

> "Brain processes involve chemical processes and hence must, in principle, be treated quantum mechanically."

We should also mention quantum field reductionist models, *Jibu* and *Yasue* [22], [23] (based on *Umezawa* [24]), *Vitiello et el.* [25]. These quantum field models look more attractive (al least for me). At the moment there is no idea how make the great jump from individual gravitational collapses to global acts

[1]We mention *Berkeley's* idealism as one of sources for such a point of view to physical reality.

of cognition. Quantum field models are more useful to provide such a global structure connecting individual quantum collapses to global acts of thinking.

However, it seems that reductionism as the general methodology of brain's study is less and less popular in cognitive sciences. After the period of large hopes associated with new possibilities to study neurons firings, there is strong disillusionment in the possibility of some physical reduction of mental processes. This is one reason for quite strong critical attitude against quantum models in cognitive sciences. In the extreme form this criticism is expressed in the following form: *"The only common thing between quantum and mental is that we have no idea how to understand any of these phenomena."* Other thing that induces prejudice against quantum-reduction theories among neurophysiologists is that quantum micro description contains many parameters that magnitudes are far from magnitudes of corresponding brain's parameters (e. g. temperature, time scale and so on). Thus creators of all quantum reductionist models of brain's functioning become immediately involved in hard battles with these parameters (e. g. high temperature of brain). Of course, it may be that all these parameter-problems are just technical temporary problems. It may be that in future even the *decoherence problem*, see, for example, [15], would be solved. Nevertheless, there are doubts about the possibility of the direct application of quantum *physical* theory to cognitive phenomena.

My critical attitude with respect to traditional quantum cognitive models is merely based on the impossibility to explain the transition from quantum processes in microworld to cognitive processes. The latter processes seems to be performed by macroscopic neural structures. And neurophysiological experience gives the strongest arguments in favour of macro neural cognition. I had numerous discussions with authors of various quantum reductionist cognitive models. Unfortunately, nobody has no idea about the possibility of such a transition from quantum micro to cognitive macro.

Finally, we discuss the holistic approach to cognitive phenomena based on Bohmian-Hiley-Pilkkänen theory of active information. By considering the pilot wave as a kind of information field they presented interesting models of cognitive processes, see [26]–[28], see also author's work [29]. In the latter paper there was proposed a mathematical model of *field of consciousness*. This field in not defined on physical space-time. This is a pure information structure. In principle, such a field can be considered as a mathematical representation of *Whitehead's field of feeling* [1].

Consciousness-information models also were developed in books of *M. Lockwood* [30], and *J. A. Barrett* [31] (who use a many-minds version of many-worlds interpretation of quantum mechanics) and author's paper [32] devoted to quantum information reality.

Last few years I try to split, see [33]–[36], the quantum formalism into two more or less independent parts:

1) really *quantum* (quanta, Planck constant, discreteness);

2) Hilbert space *probabilistic formalism*.

Pioneer investigations of *M. Planck* and *A. Einstein* on foundations of quantum theory (black body radiation and photoelectric effect) were merely investigations on discreteness (quantization) of energy. Quantum probabilistic (Hilbert space) formalism was developed later (*Born, Jordan, Heisenberg, Dirac* [37]–[38]). It was created to describe statistics of elementary particles. Due to such a historical origin, the Hilbert space probabilistic calculus is always related to processes in microworld.

However, careful analysis, [33]–[36], demonstrated that Hilbert space probabilistic calculus (*Born, Jordan, Heisenberg, Dirac,* see e. g. [37]–[38]) is a purely mathematical formalism that gives the possibility to work with context depending probabilities, i. e., probabilities depending on complexes of physical conditions (contexts) related to concrete measurements. Therefore we could apply the Hilbert space probabilistic formalism, *quantum-like formalism,* not only to the description of statistical micro phenomena, but also to various phenomena outside micro world. One of such possibilities is to apply quantum-like formalism to describe statistical experiments with cognitive systems. Here a quantum-like formalism describes probabilistic distributions depending on neural, cognitive and social contexts.

Such an approach has no (at least direct) relation to reductionist quantum models. We are not interested in statistical behaviour of micro systems forming a macro system, brain. Therefore this approach does not induce such a problem as the transition from micro to macro (temperature, decoherence and so on). We just use the Hilbert space probabilistic formalism to describe cognitive measurements. As in the ordinary quantum formalism, mental observables are realized as symmetric operators in the Hilbert space of square integrable functions $\phi(q)$ depending on the mental state q of a cognitive system. By using the Hilbert space scalar product we calculate averages of mental observables. Of course, this cognitive model is the purely statistical one. It could not provide a description of individual thought-trajectories.

One of reasons for using quantum-like formalism to describe statistics of measurements over cognitive systems is that cognitive systems (as well as quantum) are very sensitive to changes of contexts of experiments — complexes of physical and mental conditions ([33]–[36], compare to *Heisenberg* [38] or *Dirac* [37]). Therefore quantum-like formalism can be used to describe external measurements (in neurophysiology, psychology, cognitive and social sciences) over ensembles of cognitive systems or neural ensembles in a single brain. As well as in quantum experiments with elementary particles, preparation of a statistical ensemble (of rats or people) plays the crucial role in cognitive measurements. Thus, as in ordinary quantum theory, it is meaningless to speak about a measurement without to specify a preparation procedure preceding this measurement. In cognitive sciences we also should follow to *Bohr's* recommendation to take into account the whole experimental arrangement. The main experimental evidence of quantum-like structure of statistical data obtained in neurophysiology, psychology, cognitive and social sciences should be interference of probabilities, see [33]–[36] and section 10.

Moreover, our quantum-like formalism can be used not only for describing of external cognitive experiments, but also modeling of mentality. The basic assumption of our model is that brain has the ability to "feel" probabilistic amplitude $\phi(q)$ of information states produced by hierarchic neural pathways in brain (and the whole body). There is also presented a model of consciousness that creates its context by performing self-measurements over extremely sensitive neural contexts.

One of the fundamental problems in foundations of cognitive quantum-like formalism is the choice of a mathematical model for a mental configuration space on that wave function is defined. We shall discuss this problem in the details in section 2. We now only remark that the *Euclidean* physical space (in that the physical brain is located) does not look attractive as a model of mental space. Instead of this conventional model of space, we develop cognitive quantum-like formalism on the space of information strings that could be performed by chains of hierarchically ordered neurons. Such a configuration space is geometrically represented by a hierarchic p-adic tree. In fact, this idea was already discussed in authors's paper [32] (see also [39]–[44]). However, in [32] we did not use the standard Hilbert space formalism. It was used a generalization of quantum probabilistic calculus based on p-adic probabilities. In the present paper we use the standard Hilbert space formalism on p-adic trees. In fact, the mathematical formalism of p-adic quantum mechanics is well developed, see *Vladimirov, Volovich, Zelenov* [45], [46], see also [47]. We "simply" apply this formalism to cognitive phenomena.

In the ordinary quantum mechanics, we could go beyond the statistical application of quantum formalism. One of the most attractive possibilities is to use the pilot wave Bohmian formalism. As we have already remarked, the idea to use Bohmian mechanics in cognitive sciences was already well discussed (*Bohm-Hiley-Pilkkänen* [26]–[28] and author [29]). It is rather surprising that it seems to be impossible to create a variant of the pilot wave extension of quantum-like mental formalism presented in this paper. Formally we can introduce quantum-like mental potential and force. However, there is no possibility to derive the equation of motion (a kind of Newton equation) that would describe trajectories of individual mental states (describe "flows of mind"). In our formalism this is a consequence of the mathematical structure of the model. However, it may be that there are some deep cognitive features behind this mathematical result.

We start with some preliminary considerations on the choice of the geometry of a mental space.

2. Where is consciousness located?

The problem of location (or nonlocality) of consciousness (as well as more primitive cognitive processes) is widely discussed in philosophic, neurophysiological and psychological literature, see e. g. [48]–[56]. There is large variety of views starting with such a primary question:

"Does consciousness located in human brain?"

Both philosophic and neurophysiological discussions are, in fact, related to one fixed geometry, namely the Euclidean one. It seems that such an approach was originated (at least in philosophy) by *Kant* [55]. For him, the space was the absolute Euclidean space. He also pointed out that the idea of space is the primary idea. Nothing could be even imagine without any relation to space. As space is identified with the Euclidean space, we have to look for a place of consciousness in this space. It seems that this is the starting point of the main stream of modern philosophic, neurophysiological and psychological investigations. However, despite enormous efforts to find the place of consciousness, there are more and more evidences that consciousness could not be located in physical space. What is wrong? I think the choice of geometry. I think that the use of the Euclidean geometry is not adequate to this problem.

In fact, the idea that different natural phenomena are in general described by using different geometries is well established in physics, especially general relativity and string theory. Following to *Chalmers* [56], we consider consciousness as a kind of natural phenomena. First we must find an adequate model of a *mental space*. Then we get the possibility to describe cognitive (and conscious) phenomena. Let us imagine that we would like to describe electromagnetic processes without to use a mathematical model of the electromagnetic field distributed on the Euclidean space. It seems to be impossible [2].

We have already mentioned the use of various geometries in general in physics, e. g. in general relativity and string theory. However, these models are mainly locally-Euclidean (Euclidean manifolds) [3]. The use of such manifolds could not solve the problem of cognitive nonlocality (in particular, nonlocality of psychological functions). One of possibilities is to proceed in quantum-like way and use noncommutative mental coordinates, see *B. Hiley* [28]. Another possibility is to try to find a model of classical mental configuration space (probably as the basis of a quantum-like model). Since [39]–[46], we use purely information model of mental space, namely the space of all possible information strings that could be produced by hierarchically ordered chains of neurons. One of the simplest models of such a space is a hierarchic (homogeneous) p-adic tree $\mathbf{Z}_p$, where p is a natural number. It gives the number of branches leaving each vertex of this tree. We remark that in mathematical models p is typically a prime number, see [45], [47]. But it is not so important for our cognitive considerations.

3. Classical mental states produced by one-layer brain

3.1. p-adic coding. We consider the simplest hierarchic "brain" consisting of just one hierarchic chain of neurons :

$$\mathcal{N} = (n_0, n_1, \ldots, n_N, \ldots).$$

In a mathematical model it is convenient to consider an infinite chain.

In the simplest model each neuron can perform only one of two states: $\alpha_j = 1$ (*firing*) and $\alpha_j = 0$ (*off*).

In more complex models each neuron n_j can perform p different levels of activation: $\alpha_j = 0, 1, \ldots, p-1$. For example, such a coding can be obtained

[2] Sometimes (especially in philosophy) there are used words "explain consciousness". I do not think that we could "explain" it. In the same way we could not "explain" e. g. electromagnetic field. We could only describe mathematically and via such a description understand, compare to *Penrose* [15], p. 419.

[3] Even the use of superspace in superstring theory as well as in superfield theory cannot be considered as a fundamental change of geometry. Locally superspace is still a real continuous manifold, see e. g. [57] for the details.

by using frequencies of firing of neurons as basic elements of coding. Frequencies of firing are a better basis for the description of processing of information by neurons than a simple on/off. This has been shown to be the fundamental element of neuronal communication in a huge number of experimental neurophysiological studies (see e. g. [58], [59] on mathematical modeling of brain functioning in the frequency domain approach).

One of possible p-adic coding models is the following one. A p-adic structure associated with frequency coding is generated in the following way. There exists some interval (of physical time) Δ (unit of "mental time", see section 10 for further consideration). Then α_j is equal to the number of oscillations of the neuron n_j (in the hierarchic chain $\mathcal{N}$) that are performed during the interval Δ. Here $p-1$ (where $p = p_\Delta$) is the maximal possible number of oscillation during the period Δ that can be performed by neurons in the chain $\mathcal{N}$. Thus in our model the p-adic structure of brain of a cognitive system τ (that uses a frequency neural code) is related to time scale of functioning of brain, see also section 10.

It must be mentioned one mathematical fact that may be have some cognitive interpretation. The case $p = 2$ is the very exceptional one in p-adic analysis, see e. g. [45], [47]. We can speculate that the transition from 2-adic coding (firing/off) to more complex p-adic, $p > 3$, coding (e. g. frequency coding) was the evolutionary jump. Cognitive systems in the p-adic model exhibit essentially reacher mental behaviour in the case $p > 2$ than in the case $p = 2$, see [47], [39]–[44] on classical mental dynamics.

3.2. Hierarchy and ultrametricity. It is supposed that neurons in a layer $\mathcal{N}$ are hierarchically ordered: n_0 is the most important (igniting), n_1 is less important and so on. The $\mathcal{N}$ is able to produce information strings of the form:

$$x = (x_0, x_1, \ldots, x_N, \ldots), \quad x_j = 0, 1, \ldots, p-1.$$

We denote the set of all such strings by the symbol $\mathbf{Z}_p$. The hierarchic structure in the chain $\mathcal{N}$ induces a tree representation of $\mathbf{Z}_p$. Information strings are represented by branches of such a tree.

The distance between two branches, x and y, is defined in the following way. Let l be the length of the *common root* of these branches. Then the p-adic distance between x and y is defined as

$$\rho_p(x, y) = \frac{1}{p^l} \, .$$

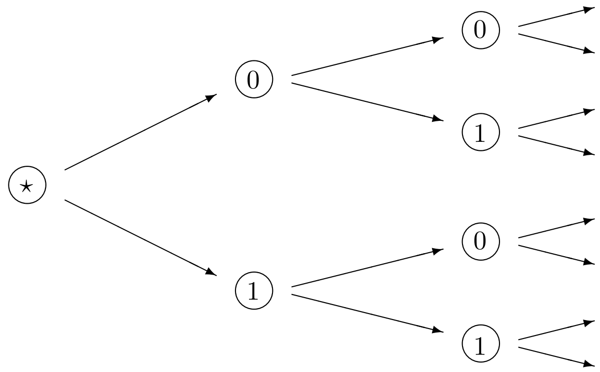

Figure 1. The 2-adic tree

Thus if $x = (x_j)$ and $y = (y_j)$ and $x_0 = y_0, \ldots, x_{l-1} = y_{l-1}$, but $x_l \neq y_l$, then $\rho_p(x, y) = (1/p^l)$. This is a metric on the set of branches $\mathbf{Z}_p$ of the p-adic tree. Two branches all close with respect to this metric if they have sufficiently long common root. We remark that $\mathbf{Z}_p$ is complete with respect to the p-adic metric ρ_p.

The p-adic metric gives a topological representation of the hierarchic structure in neural chains. The distance between information strings x and y approaches the maximal value $\rho_p(x, y) = 1$ if $x_0 \neq y_0$. Thus the state (e. g. the frequency of firing) of the first neuron n_0 in a hierarchic chain $\mathcal{N}$ plays the most important role. States x_j of neurons n_j, where $j \to \infty$, have practically negligible contribution into geometry of the p-adic space.

The p-adic metric is so called *ultrametric,* i. e., it satisfies the *strong triangle inequality*:

$$\rho_p(x, y) \leq \max\bigl[\rho_p(x, z), \rho_p(z, y)\bigr], \quad x, y, z \in \mathbf{Z}_p\,.$$

The strong triangle inequality can be stated geometrically: *each side of a triangle is at most as long as the longest one of the two other sides.* This property implies that all triangles are isosceles. Ultrametricity is the very important feature of p-adic geometry. In fact, ultrametricity is the exhibition of hierarchy. Recently it was proved in general topology that in general case ultrametricity induces a treelike representation and vice versa, see [60]. In many particular cases such a relation between ultrametricity and hierarchy was used in theory of spin glasses, see e. g. [61]–[63].

There exists a natural algebraic structure on this tree: addition, subtraction and multiplication of branches. It is based on the representation of information strings by so called p-adic numbers:

$$x = x_0 + x_1 p + \ldots + x_N p^N + \ldots$$

This is the ring of p-adic integers. In particular, this is compact additive group. Thus there exists the Haar measure dx (an analogue of the ordinary linear measure on the straight line).

We set $B_r(a) = \{x \in \mathbf{Z}_p : \rho_p(x, a) \leq r\}$ and $S_r(a) = \{x \in \mathbf{Z}_p : \rho_p(x, a) = r\}$, where $r = 1/p^j, j = 0, 1, 2, \ldots$ and $a \in \mathbf{Z}_p$. These are, respectively, balls and spheres in the metric space $\mathbf{Z}_p$. In particular, $\mathbf{Z}_p = B_1(0)$. Each ball has the structure of the homogeneous p-adic tree (scaling of the basic tree given by $\mathbf{Z}_p$).

As in every ulrametric space, all these sets (balls and spheres) have a topological structure which seems to be rather strange from the point of view of our Euclidean intuition: they are open and closed at the same time. Such sets are called *clopen.* Another interesting property of p-adic balls is that two balls have nonempty intersection iff one of these balls is contained in another. Finally we note that any point of the p-adic ball can be chosen as its center. Thus the ball is not characterized by its center and radius.

3.3. Mental space. We choose the space $Q = \mathbf{Z}_p$ as a *mental configuration space.* Points $q \in Q$ are called mental classical-like states (or simply *mental states*) or mental positions.

Thus a mental state $q \in Q$ describes activity of neurons in a hierarchically ordered chain of neurons. This is a kind of information state. Such a state could not be considered simply as the representation of physical (electro–chemical) activity of neurons in a chain. There are two information parameters that play important roles in our model.

First there is the hierarchic structure in a neural chain. Neurons in a chain "do not have equal rights." The igniting neuron n_0 is the bandmaster of the orchestra $\mathcal{N}$. This orchestra is rigidly hierarchic. The next neuron n_1 in the $\mathcal{N}$ is less important than n_0 and so on. I think that the presence of such a hierarchy plays an important role in creation of cognition and may be even consciousness.

Another information parameter is a natural number p that determines the coding system of (one layer) brain $\mathcal{N}$. If we follow to the frequency approach to functioning of neural networks in brain, then the parameter p gives the maximal number of oscillation for a neuron in a chain $\mathcal{N}$ during the unit interval Δ of *mental time.* The Δ is an interval of physical time that in our model determines the neural code of $\mathcal{N}$, see section 10.

In our model a mental state provides only *cognitive representation* and not the *contents of consciousness.* All unconscious processes are performed on the level of mental states. We remark that in a multi-layers brain, see section 5.1, there can be performed parallely various unconscious cognitive processes. Nonlinear dynamical models of such processes were studied in [47], [39]–[44]. One of distinguishing features of p-adic nonlinear dynamics is the absence of chaotic behaviour. In general p-adic dynamical systems are essentially more regular than real ones. Moreover, they are very stable with respect to random perturbations (in particular, noises), [43]. Typically a p-adic random dynamical system has only deterministic attractors, see [43]. We remark that dynamics in spaces of p-adic numbers depends crucially on the parameter p (determining the neural code of a brain $\mathcal{N}$). As it was discovered in [47], the same dynamical system (e. g. given by a monomial x^n) can demonstrate completely different behaviours for e. g. $p = 2$, or $p = 3$, or $p = 1999, \ldots$

We are now going to consider a quantum-like model based on the p-adic mental configuration space $Q = \mathbf{Z}_p$. In particular, this model might be used to describe the transition from unconscious representation of cognitive information to conscious one.

4. Quantum-like formalism for one layer brain

4.1. Hilbert space probabilistic formalism for mental observables. We consider the space of square integrable functions $L_2(Q, dx)$, where $Q = \mathbf{Z}_p$:

$$\phi : \mathbf{Z}_p \to \mathbf{C}, \quad \|\phi\|^2 = \int_{\mathbf{Z}_p} |\phi(x)|^2 dx < \infty \, .$$

The space $\mathcal{H} = L_2(Q, dx)$ is chosen as the space of *mental quantum-like states* (or *mental amplitudes*). These states are represented by normalized vectors $\phi \in \mathcal{H} : \|\phi\| = 1$. The $\mathcal{H}$ is a complex Hilbert space with scalar product

$$(\phi, \psi) = \int_Q \phi(x)\bar{\psi}(x)dx \, . \qquad (1)$$

Mental observables are realized as self-adjoint operators $A : \mathcal{H} \to \mathcal{H}$. As in the ordinary quantum formalism, by fixing a quantum-like state $\phi \in \mathcal{H}$ in general we do not fix the concrete value $A = \lambda$ of a mental observable A. It is only possible to find the average of A in the state ϕ :

$$\langle A \rangle_\phi = \int_Q A(\phi)(x)\bar{\phi}(x)\, dx \; . \tag{2}$$

However, if $\phi \in \mathcal{H}$ is an eigenfunction of A corresponding to the eigenvalue λ, i. e., $A\phi = \lambda\phi$, then we can be sure that we shall obtain the value $A = \lambda$ with probability 1.

The concrete representations of mental observables by self-adjoint operators is very important and nontrivial problem. This problem could not be solved by trivial generalization of ordinary quantum formalism. We start with the surprising remark: it seems to be *impossible to define mental position,* q, observable. Formally the difficulty is purely mathematical: we could not multiply a p-adic number $q \in Q$ with a complex number $\phi(q)$. Therefore the standard Shrödinger's definition of the position operator could not be generalized to the cognitive case. Of course, we could try to find some mathematical tricky ("non natural") definitions of mental position operator. However, it might be that this mathematical difficulty is an evidence of some important feature of cognitive systems. It might be that

Even in principle it is impossible to measure mental states Q of brain.

In particular, we could not prepare brain in the fixed mental state (there are no mental state eigenfunctions).

We can only find the probability that mental state q belong to some (measurable) subset O of the mental space $Q : P(q \in O) = \int_O |\phi(x)|^2 dx$.

Example 4.1. Let us consider the quantum like state $\phi \equiv 1$ (the uniform probability distribution of mental states). Then $P(q \in B_r(a)) = r$. Thus (as it could be expected) the probability to find this cognitive system in the mental state q belonging to a small ball around any fixed point a is small.

4.2. External mental measurements. An important class of mental observables is given by measurements that are performed by external systems over a cognitive system τ. In particular, in section 6 we shall introduce neuron-activation observable that arises naturally in neurophysiological measurements. Besides neurophysiological mental observables, we can consider e. g. psychological or social mental observables. In experiments with people such a mental observable A can be given just by a question A. Here A takes two values: $A = 1$, *yes*, and $A = 0$, *no*. In experiments with animals values of A give possible reactions of animals to experimental conditions. In principle an external system that performs a measurement over a cognitive system τ need not be conscious nor even cognitive. It can be, for example, a magnetic resonance device performing a measurement of neural activity [4].

4.3. Consciousness. I would not like to reduce mental measurements to external measurements. It is natural to try to describe consciousness as a continuous flow of mental self-measurements [5]. The idea that cognitive representation of information in brain becomes conscious in process of self-measurements is not so new, see e. g. *Orlov* [13] for a quantum logic model of self-measuring consciousness:

> "... the volitional act of a free choice plays in this theory a role analogous to the role of the measurement act in quantum mechanics (with the important difference that the brain "measures" itself). Consciousness is a system which observes itself and evaluates itself – being aware, at the same time, of doing so."

The crucial point of our consideration is that we use quantum-like ideology, instead of the traditional quantum one. In our model the configuration space is the state space of macroscopic neural networks. Thus we need not go deeply into microworld to find the origin of consciousness (e. g. no collapses of mass-superpositions and so on). So we need not apply to quantum gravity (or even superstring theory).

In our model it is supposed that each cognitive (at least conscious) system τ developed the ability to feel the probability distribution $P(q)$ of realization of the hierarchic information string q by its neural system. Such an ability is basically transferred from generation to generation. However, for each τ it is permanently developed in the process of brain's functioning. This probability distribution $P(q)$ has an amplitude $\phi(q)$ that can be mathematically described by a normalized vector in the Hilbert space $\mathcal{H} = L_2(Q, dx)$. As usual, $P(q) = |\phi(q)|^2$. As it was already discussed the appearance of the quantum-like probabilistic formalism (instead of classical Kolmogorov probabilistic formalism) is a general consequence of sensibility of $P(q)$ to changes in the neural context. Here

$$\phi(q) = P(q)e^{i\theta(q)}.$$

[4] We understood that such a viewpoint induces some difficulties, see, for example, *Wigner* [8] on the role of consciousness in quantum measurements.

[5] "Continuous" has the meaning of mental time continuity, see section 10.

Here $\theta(q)$ is a phase parameter. It appears automatically in transformation of probabilities from one mental representation (see 5.4) to another, see [33]–[36]. We shall illustrate the role of $\theta(q)$ on the example of transition from mental-position to motivation representation, section 5.4.

In our model "feeling" of the probability distribution is performed is performed on unconscious level. In particular a cognitive system does not feel consciously the evolution of the mental amplitude $\phi(t, q)$.

Moreover, we suppose that each conscious cognitive system τ has the ability to perform self-measurements. Results of these measurements form the contexts of consciousness. I do not try to develop such a model of consciousness in the present paper. The main aim of this paper was to present quantum-like formalism corresponding to *hierarchic* neural networks. In principle, the reader can use only restricted viewpoint to mental observables as external measurements over cognitive systems. We just consider a possible scheme of functioning of such a (quantum-like) self-measuring consciousness.

4.4. Random dynamical quantum-like consciousness. Let us denote the set of all operators representing mental observables participating in creation of the contents of consciousness by the symbol $\mathcal{L}_{\rm cons}(\mathcal{H})$. Let us consider a random dynamical system (RDS: see, for example, [64] for general theory) that at each instant of (mental) time chooses randomly some set of commutative operators $A_1, \ldots, A_m \in \mathcal{L}_{\rm cons}(\mathcal{H})$. The contents of consciousness at this instant of time is created by the simultaneous measurement of $A_1, \ldots, A_m$.

One of the main distinguishing features of the RDS-model is that a RDS in the space of mental observables can have long range memory. Such feature of RDS is very important to create a realistic mathematical model of functioning of consciousness. Our consciousness does not consist of discrete moments but there is flow of consciousness. We remember something about our earlier conscious experiences, see e. g. *Whitehead's* analysis of this problem [1], pp. 342–343:

> "Whenever there is consciousness there is some element of recollection. It recalls earlier phases from the dim recesses of the unconscious. Long ago this truth was asserted in *Plato* doctrine of reminiscence. No doubt *Plato* was directly thinking of glimpses of eternal truths lingering in a soul derivative from timeless heaven of pure form. Be that as it may, then in a wider sense consciousness enlightens experience which precedes it, and could be without it if considered as a mere datum. *Hume*, with opposite limitations to his meaning, asserts the same doctrine. . . But the immediate

point is the deep-seated alliance of consciousness with recollection both for *Plato* and for *Hume*."

5. Motivation observable

5.1. Multi-layers hierarchic brain. To consider nontrivial examples of mental observables, it is convenient to study a "brain" having more complex mental space. Such a brain consists of a few hierarchic p-adic trees. We consider a layer of neurons

$$\mathcal{N} = (\ldots, n_k, \ldots, n_0, \ldots, n_l, \ldots)$$

that goes in both directions (in the mathematical model it is infinite in both directions). Each neuron $n_j, j = 0, \pm 1, \pm 2, \ldots,$ can be the igniting neuron for right hand side hierarchic chain: $\mathcal{N}_j = (n_j, \ldots, n_l, \ldots)$. The corresponding mental space $\mathbf{Z}^{(j)}$ consists of all information strings

$$x = (x_j, x_{j+1}, \ldots, x_l, \ldots), \quad x_l = 0, 1, \ldots, p-1$$

(in particular, $\mathbf{Z}_p = \mathbf{Z}^{(0)}$). Each space has the structure of the homogeneous p-adic tree. These spaces are ordered by inclusion: $\mathbf{Z}^{(j+1)} \subset \mathbf{Z}^{(j)}$. We consider union of all these space $\mathbf{Q}_p = \cup_{j=-\infty}^{\infty} \mathbf{Z}^{(j)}$. Geometrically this space is represented as a huge collection of trees ordered by the inclusion relation. On this space we can introduce the structure of ring: addition, subtraction and multiplication of branches of trees. If the coding parameter p is a prime number (i. e., $p = 2, 5, 7, \ldots, 1997, 1999, \ldots$), then $\mathbf{Q}_p$ is a field, i. e., division of branches also is well defined. In this case $\mathbf{Q}_p$ is a number field (of p-adic numbers). Arithmetical operations are performed by using p-adic number representation of branches:

$$x = \sum_{i=j}^{\infty} x_i p^i, \ j = 0, \pm 1, \pm 2, \ldots \tag{3}$$

Metric on $\mathbf{Q}_p$ is defined in the same way as on $\mathbf{Z}_p$. In particular, each tree $\mathbf{Z}^{(j)}$ coincides with a p-adic ball $B_r(0)$, where $r = 1/p^j$. We shall also use p-adic absolute value: $|x|_p = \rho_p(x, 0)$. To calculate it, we have to find in the chain $\mathcal{N}$ the first (from the left hand side) firing neuron n_j ($x_j \neq 0$, but $x_l = 0$ for all $l < j$) and set $|x|_p = 1/p^j$.

The $\mathbf{Q}_p$ is a locally compact field. Hence, there also exists the Haar measure dx.

We now choose $Q = \mathbf{Q}_p$ as a model of a mental configuration space; consider the Hilbert $\mathcal{H} = L_2(Q, dx)$ of square integrable functions $\phi : Q \to C$ as the space of quantum-like mental states.

5.2. Motivation magnitude observable. It would be interesting to consider the following quantity (more precisely, qualia): *motivation* ξ to change the mental state q. Unfortunately, by the same reasons as for the mental state observable we could not introduce a motivation observable. However, we can introduce an observable M_ξ that will give the magnitude of a motivation. It is impossible to prepare a brain with the fixed motivation ξ, but we could prepare a brain with the fixed amplitude of a motivation (that gives a measure of motivation's strength). Such M_ξ must be a kind of derivative with respect to the mental state (coordinate) q. Such a generalization of derivative is given by *Vladimirov's operator* D, see [45], defined with the aid of the p-adic Fourier transform.[6]

p-adic Fourier transform:

$$\widetilde{\phi}(\xi) = \int_Q \phi(x) e(\xi x) dx, \quad \xi \in Q,$$

where e is a p-adic character (an analogue of exponent): $e(\xi x) = e^{2\pi i \{\xi x\}}$. Here, for a p-adic number a, $\{a\}$ denotes its fractional part, i. e., for

$$a = \frac{a_{-m}}{p^m} + \ldots + \frac{a_{-1}}{p} + a_0 + \ldots + a_k p^k + \ldots$$

(where $a_j = 0, 1, \ldots, p-1$, and $a_{-m} \neq 0$) we have

$$\{a\} = \frac{a_{-m}}{p^m} + \ldots + \frac{a_{-1}}{p}\,.$$

Vladimirov's operator of order $\alpha > 0$ is defined as

$$D^\alpha(\phi)(x) = \int_Q |\xi|^\alpha \widetilde{\phi}(\xi) e(-\xi x) d\xi\,.$$

We remark that $D^\alpha D^\beta = D^{\alpha+\beta}$. We define the motivation magnitude observable M as $M_\xi = hD$.

Here $h = (1/p^m)$ is some normalization constant. The h plays the role of the Planck constant in ordinary quantum mechanics. At the moment it is not

[6] We remark that it is impossible to define the derivative for maps from $\mathbf{Q}_p$ to $\mathbf{R}$, see [47].

clear: "Can we expect that there exists a kind of universal constant h, the *mental Planck constant*?" I am quite sceptical that such a universal normalization constant really exists. It is more natural to suppose that h would depend on a class of cognitive systems under consideration. In fact, by finding h (the level of motivation discretization) we find the basis p of the coding system.

To calculate averages of the momentum magnitude operator M_ξ for different quantum-like mental states, it is natural to use the Fourier transform. By analogy with ordinary quantum mechanics we could say: to move from position to momentum representation.

Example 5.1. Let a quantum-like state ϕ is such that its Fourier transform $\widetilde{\phi}(\xi)$ is uniformly distributed over the ball $B_r(0), r = 1/p^l$. Here

$$\langle M_\xi \rangle_\phi = p^l \int_{B_r(0)} |\xi|_p \, d\xi = \frac{1}{p^{l-1}(p+1)} \, .$$

5.3. Wholeness of mental observables. It is important to remark that (in the opposite to the ordinary quantum momentum) the M_ξ is *nonlocal operator*. It can be represented as an integral operator, see [45]:

$$D(\phi)(x) = \frac{p^2}{p+1} \int_Q \frac{\phi(x) - \phi(y)}{|x-y|_p^2} dy \, .$$

To find $M_\xi(\phi)(x)$ in some fixed point x, we have to take into account values of ϕ in all points of the mental configuration space. Thus *motivation psychological function* could not be localized in some particular neural substructure of brain.

This example is a good illustration of the mathematical description of nonlocality of psychological functions in our p-adic quantum-like model. One of the main distinguishing features of this model is nonlocality of derivation operator (Vladimirov's operator). Hence the corresponding psychological function is produced by the whole neural system of body (as indivisible system).

5.4. Psychological functions as quantum-like representations. The mathematical description of the motivation psychological function by using a new representation in the Hilbert state space is the basic example that can be generalized to describe all possible psychological functions. We remark that the motivation representation is, in fact, a new system of quantum-like mental coordinates. In the case of motivation a new system of coordinates was generated by a unitary operator in the Hilbert state space, namely Fourier transform.

In the general case each psychological function F is represented mathematically by choosing a system of coordinates in the Hilbert state space, *mental representation.* Thus we can identify the set of all psychological functions with the set of all unitary operators: $U \to F_U$ and $F \to U_F$. All mental observables A represented by self-adjoint operators that can be diagonalized by using the concrete U-representation can be related to the corresponding psychological function F_U. For example, for the visual function observables of shape and colour can be diagonalized in the visual representation of the state Hilbert space.

In such a model all psychological functions coexist peacefully in the neural system. The evolution of the quantum-like mental state $\phi(t, x)$ (see section 9) induces the simultaneous evolutions of all mental functions (in this state). This is unconscious evolution. Thus a conscious system τ are not consciously aware about simultaneous evolution of the various psychological functions. Only by the acts of self-measurements of some mental observables $A_F^{(j)}$ that are diagonal in the F-representation the τ becomes aware about some features of the corresponding psychological function F.

5.5. Free mental waves. We remark that Vladimirov's operator D has a system of (generalized) eigenfunctions that is similar to the system of free-wave eigenfunctions in ordinary quantum mechanics, where $\phi_\xi(x) = e^{i\xi x/h}$ corresponds to the fixed value ξ of momentum. In the mental framework:

$$M_\xi e(h\xi x) = |\xi|_p e(h\xi x) \,.$$

Here we have used the fact [45]: $De(\xi x) = |\xi|_p e(\xi x)$. We remark that in the ordinary quantum formalism the h is placed in denominator, $\xi x/h$, and in the p-adic quantum formalism it is placed in the nominator, $h\xi x$. This is a consequence of the fact that $1/h$ is large in $\mathbf{R}$ and h is large in $\mathbf{Q}_p$.

The function $\phi_\xi(x) = e(h\xi x)$ is a kind of *free mental wave* corresponding to the fixed value ξ of the motivation. As $|\phi_\xi(x)| = 1$ for all $x \in Q$, the probability to find a cognitive system in the mental state x does not depend on x. By analogy with the ordinary quantum mechanics we would like to interpret this mathematical fact in the following way: By fixing the magnitude of motivation (strength of willing) we could not localize the mental state. However, we see soon that such an analogy (between material and mental states) could not be used.

A free mental wave ϕ_ξ gives a good example illustrating the role of the phase $\theta(x)$ of the mental amplitude. Here we have

$$\theta(x) = 2\pi\{\xi x\}. \tag{4}$$

Thus if a cognitive system τ has the fixed motivation ξ and the mental probability distribution $P(x)$ is uniform, then the phase of the corresponding mental amplitude is determined by (4). Thus in general the phase $\theta(x)$ of a mental amplitude $\phi(x)$ is not the pure product neural activity. This phase contains information on transition form one mental representation to another.

5.6. Privacy of motivation states. The wave $\phi_\xi(x)$ is not determined uniquely by the observable M_ξ. The main distinguishing feature of p-adic quantum mechanics (discovered by *Vladimirov*, [45]) is huge degeneration of spectrum of the momentum and energy operators. In particular, beside eigenfunctions $\phi_\xi(x)$, the M_ξ has an infinite set of other eigenfunctions corresponding to the eigenvalue $\lambda = |\xi|_p (= p^k$ for some $k = 0, \pm 1, \pm 2, \ldots)$.

Each $\lambda = p^k, k = 0, \pm 1, \pm 2, \ldots$ corresponds to an infinite series of eigenfunctions (distinct from the free mental wave $\phi_\xi(x)$) belonging to $L_2(Q, dx)$ [7]. These eigenfunctions are well localized (concentrated in balls) in the mental configuration space.

This is very natural from the mental point of view. It would be quite strange if the only quantum-like mental state with the fixed motivation magnitude is the state ϕ_ξ characterized by totally indefinite distribution of mental states q. By intuitive reasons there must be quantum-like mental states characterized by the fixed $M_\xi = \lambda$ that are concentrated on a special class of mental states (a kind of special mental activity).

One of the most important distinguishing features of quantum-like mental theory is that the motivation magnitude operator M_ξ has discrete spectrum (except to one point, see further considerations). Hence the magnitude of the motivation could not change continuously.

There exists only one point of spectrum of the operator M_ξ that is not its eigenvalue: $\lambda = 0$. It is the limit point of the eigenvalues $\lambda_k = p^k, k \to \infty$. There is no eigenfunction ϕ_0 belonging to the state space $\mathcal{H}$. Thus in our model

[7] We remark that free mental waves $\phi_\xi(x)$ are so called generalized eigenfunctions. They are not square integrable. Thus they do not belong to the space of quantum-like mental states $\mathcal{H} = L_2(Q, dx)$. One could speculate that such non-normalizable free mental waves may be related to altered consciousness events such as e. g. hallucinations.

brain could not be (alive, awake?) in the stationary quantum-like mental state having the motivation of zero magnitude.

Another distinguishing feature is infinite degeneration of spectrum. This purely mathematical result can have important implications for the problem of *correspondence between mental and physical worlds.* In fact, due to this huge degeneration, *we could not uniquely determine the mental state of a cognitive system by fixing the motivation magnitude* M_ξ.

6. Neuron-activation observable

As we have already discussed, we could not introduce a mental state observable q. However, in the same way as for the motivation we can introduce an operator of the p-adic magnitude of a mental state:

$$M_q\phi(x) = |x|_p\phi(x).$$

Spectral properties of this operator are similar to spectral properties of the operator M_ξ : discreteness and infinite degeneration of spectrum. Eigenfunctions of M_q (belonging to $\mathcal{H} = L_2(Q, dx)$) are localized in p-adic balls–trees. Therefore:

There exist stationary states of M_q *that are characterized by activation of the fixed tree of mental states.*

Unfortunately, M_q could not be used to fix such a tree (as a consequence of infinite degeneration of spectrum).

The operators of position and motivation magnitudes, M_ξ and M_q, do not commute (as operators of position and momentum in ordinary quantum mechanics):

$$[M_q, M_\xi] = M_q M_\xi - M_\xi M_q = hJ,$$

where $J \neq 0$ is an integral operator [45]. Thus we get a *mental uncertainty relation*, compare to [32]:

For any quantum-like mental state ϕ, *it is impossible to measure motivation and position magnitudes with an arbitrary precision.*

By measuring the motivation magnitudes we change position magnitudes and vice versa. This can also be expressed mathematically by using the p-adic Fourier transform. We denote by $\Omega_r(x)$ the characteristic function of the ball $B_r(0)$ (it equals to 1 on the ball and 0 outside the ball). We have [45], p.102,

$$\widetilde{\Omega}_r(\xi) = \frac{1}{r}\Omega_{\frac{1}{r}}(\xi)\,.$$

If the state of mind is concentrated on the ball-tree $B_r(0)$, then motivations are concentrated on the ball-tree $B_{\frac{1}{r}}(0)$.

As in the case of the M_ξ-observable, the point $\lambda = 0$ belongs to non discrete spectrum of the M_q observable. Thus there is no stationary quantum-like mental state ϕ corresponding to zero magnitude of q. A cognitive system is not alive (awake?) in such a state.

To understand better the mental meaning of the M_q-observable, it is useful to consider a new mental observable:

$$A = -\log_p M_q \, .$$

If, $\phi \in \mathcal{H}$ is an eigenstate of the M_q corresponding to the eigenvalue $\lambda = |q|_p = (1/p^k)$, then ϕ also is an eigenstate of A corresponding to the eigenvalue $\mu = k$ and vice versa. Thus the discrete part of the A-spectrum coincides with the set of integers $\mathbf{Z}$. The A gives the position of the igniting neuron in a layer of neurons. It is called *neuron-activation observable.* We note that there is an interesting relation between neuron-activation observable and entropy.

Let us consider the quantum-like state $\phi(q) = \sqrt{(p+1)|q|_p}\Omega_1(q)$. Here $\sqrt{p+1}$ is just the normalization constant. The corresponding probability distribution $\mathbf{P}(q) = (p+1)|q|_p$ on the tree $\mathbf{Z}_p$ and equals to zero outside this tree. The entropy of this probability distribution

$$E_{\mathbf{P}} = -\int_{\mathbf{Z}_p} \log_p \mathbf{P}(q)\mathbf{P}(q)dq = \langle A \rangle_\phi - \log_p(p+1)\, .$$

7. Complex cognitive systems; evolution

We now consider a cognitive system consisting of n hierarchic layers of neurons. It can be an individual brain as well as a system of brains. Mental space of this cognitive system is

$$Q = \mathbf{Q}_p \times \cdots \mathbf{Q}_p$$

(n times). For each mental coordinate $q_j, j = 1, 2, \ldots, n$, we introduce the motivation magnitude operator $M_j = hD_j$, where D_j is Vladimirov operator for q_j. We introduce *kinetic mental energy* (free energy of motivations) as

$$H = h^2 \Delta\, ,$$

where $\Delta = \sum_{j=1}^n D_j^2$ is *Vladimirovian* (a p-adic analogue of the Laplacian).

We note that free mental waves $\phi_\xi(x) = e(h\xi x)$ are eigenfunctions of this operator with eigenvalues $\lambda = |\xi|_p^2$. As in the cases of the M_q, M_ξ-observables, there is an infinite family of other eigenfunctions distinct from free mental waves. These functions are localized on the mental configuration space (describing fixed ranges of ideas). Spectrum is discrete: $\lambda = p^k, k = 0, \pm 1, \pm 2$. Thus the kinetic mental energy is changed only by jumps. The $\lambda = 0$ is the only point that belongs to the non discrete spectrum of the operator of the kinetic mental energy.

Interactions between brain's layers as well as interactions with the external world are described by the operator of the potential mental energy. It is given by a real valued function (potential) $V(q_1, \ldots, q_n)$. The total mental energy is represented by the operator:

$$H = h^2 \Delta + V \, .$$

We note that a mental potential $V(q_1, \ldots, q_n)$ can change crucially spectral properties of the mental energy observable. If V depends only on p-adic magnitudes $|q_j|_p$ of mental coordinates and $V \to \infty, |q_j|_p \to \infty$, and V is bounded from below (e. g. nonnegative), then spectrum of H (that is discrete) has only finite degeneration. Thus the "state of mind" of a free cognitive system could not be determined by fixing the mental energy. However, by using additional mental (information) potentials we could (at least in principle) do this.

The ground mental energy state λ_0 is not degenerated at all. In the latter case by fixing the minimal value of the mental energy $H = \lambda_0$ we can determine the "state of mind", namely the λ_0-eigenstate. Even for other eigenvalues we can try to determine the "state of mind" if the degeneration of spectrum is not so large. It is interesting to remark that mathematical results [45] imply that degeneration of eigenvalues (distinct from the ground energy) increases (as p^2) with increasing of p. If we connect the complexity of a cognitive system with the coding base p, then we obtain that, for complex cognitive systems (e. g. $p = 1999$), it is practically impossible to determine the "state of mind" corresponding to the fixed value of mental energy.

8. Entanglement of psychological functions

8.1. Classical viewpoint to localization of psychological functions. The problem of neural localization of psychological functions split neurophysiological community, see e. g. *A. R. Damasio* [53]:

> "One held that psychological functions such as language or memory could never be traced to a particular region of brain. If one had to accept, reluctantly, that the brain did produce the mind, it did so as a whole and not as a collection of parts with special functions. The other camp held that, on the contrary, the brain did have specialized parts and those pars generate separate mind functions."

Both adherents of wholeness and localization of psychological functions have a lot of experimental evidences supporting their views. A kind of peaceful unification of these two views to localization of psychological functions is given by our model of coding of cognitive information by hierarchic pathways activity. As each pathway $\mathcal{N}$ is hierarchic, then there are a few neurons in the pathway that play the most important role. Their location in some domain U of brain determines localization of a psychological function containing $\mathcal{N}$. However, $\mathcal{N}$ goes throughout many other brain (and body) regions. So U-localization is only a kind of *fuzzy localization.*

However, I do not think that this is the end of the localization story. We suppose that cognition involves not only classical dynamics of neural networks, but also quantum-like processing described by the evolution of quantum-like wave function, see section 9. The latter gives the amplitude of probability distribution of realization of classical mental states (hierarchic strings of e. g. frequencies of firings). Such a quantum-like processing of cognitive information would automatically create psychological functions that do not have even fuzzy localization. Such functions are induced via entanglement of localized psychological functions.

8.2. Entanglement. Let $U_1, \ldots, U_k$ be some neural structures — ensembles of hierarchic neural pathways — specialized on performing psychological functions $F_1, \ldots, F_k$. Consider corresponding Hilbert spaces of quantum-like mental states: $\mathcal{H}_j = L_2(Q_j, dx_j)$, where $Q_j = \mathbf{Q}_p{}^{n_j}$ and dx_j is the Haar measure on Q_j. Let e_{F_j} be the orthonormal basis in $\mathcal{H}_j$ corresponding to the function F_j.

Let us consider the Hilbert space of quantum-like mental states of the composite neural system, $U = U_1 \cup \ldots \cup U_k$: $\mathcal{H} = L_2(Q, dx)$, where $Q = Q_1 \times \ldots \times Q_k$. Here a normalized state $\phi(q_1, \ldots, q_k)$ gives the amplitude of probability that U_1 produces q_1, ..., U_k produces q_k. We consider in $\mathcal{H}$ the orthonormal basis e obtained as the tensor product of bases e_{F_j}. The e describes a mental representation corresponding to the psychological function $F = (F_1, \ldots, F_k)$ produced by classical combination of psychological functions $F_1, \dots, F_k$. Let us now consider some other basis $\tilde{e}$ containing nontrivial linear combinations of vectors of the e. This basis gives the mental representation of a psychological function G that could not be reduced to classical combination of psychological functions F_j. We call G an *entanglement of psychological functions* F_j. Of course, G is produced by the collection U of neural structures U_j. But G arises as nontrivial quantum-like combination of psychological functions F_j.

We remark that entanglement of psychological functions has nothing to do with entanglement of quantum states of individual micro systems in brain (compare to conventional reductionist quantum models of brain functioning). Entanglement of psychological functions is entanglement of probabilistic amplitudes for information states of macroscopic neural systems, see section 10 for further discussion.

9. State-evolution

We want to describe the evolution of a quantum-like mental state (mental wave function) $\phi(t,x)$. The first natural and rather nontrivial problem is the choice of the evolution parameter t. This problem was discussed in the details in [32]. It was shown that there are different natural possibilities to describe the evolution of mental states: "mental time", "psychological time" as well as ordinary physical time evolution, see also section 10. In this paper we consider the evolution with respect to physical time t belonging to the real line $\mathbf{R}$. To derive the evolutional equation for $\phi(t,x)$, we proceed in the same way as Schrödinger in ordinary quantum mechanics. We start with a free mental wave $\phi_\xi(x) = e(h\xi x),\ \xi, x \in \mathbf{Q}_p$. We have:

$$H_0 \phi_\xi(x) = |\xi|_p^2 \phi_\xi(x)\,,$$

where $H_0 = h^2 D^2$ is the operator of the mental energy for a free system.

The $\phi_\xi(x)$ is a stationary state corresponding to mental energy $E = |\xi|_p^2$. Such a wave evolves as

$$\phi_\xi(t,x) = e^{\frac{iEt}{h}} \phi_\xi(x)\,.$$

We note that this function is a combination of two essentially different exponents: ordinary exponent and p-adic character. This function satisfies to the evolutional equation:

$$ih\frac{\partial \phi}{\partial t}(t,x) = h^2 D^2 \phi(t,x)\,. \tag{5}$$

This is *Schrödinger's mental equation* for a free cognitive system. If we introduce a mental potential $V(x)$, then we get general Schrödinger's mental equation:

$$ih\frac{\partial\phi}{\partial t}(t,x) = h^2 D^2\phi(t,x) + V(x)\phi(t,x)\,. \tag{6}$$

If the initial quantum-like state $\psi(x) = \phi(0,x)$ is known, then by using (6) we can find $\phi(t,x)$ at each instant t of physical time. Under quite general conditions [45], the operator $H = h^2D^2 + V(x)$ is a self-adjoint operator. Therefore (6) is standard Schrödinger's equation in the Hilbert space $\mathcal{H}$ for one rather special class of operators H. There also are mathematical results on analytical properties of solutions and correctness of Cauchy problem [47].

Remark 9.1. (Bohmian theory) We can try to develop an analogue of Bohmian (pilot wave) approach. As in ordinary Bohmian mechanics, we can define a quantum-like mental potential

$$W_\phi(t,x) = -\frac{h^2}{R}D^2R, \quad \text{where } R(t,x) = |\phi(t,x)|\,. \tag{7}$$

This potential has the same properties as the ordinary quantum potential:

(a) $W_\phi(t,x)$ does not depend on the absolute magnitude of ϕ;

(b) $W_\phi(t,x)$ depends on the second variation of the magnitude of ϕ.

However, (in the opposite to ordinary Bohmian mechanics) we could not describe evolution of an individual mental state (position) $q(t)$ by using Newton's equation with additional potential W. From the first sight this is a purely mathematical difficulty. But I think that this mathematical fact has deep cognitive meaning, namely that dynamics of quantum-like state $\phi(t,x)$ does not determine dynamics of classical mental states. Very different flows of classical mental states (hierarchically ordered neural flows) can produce the same wave $\phi(t,x)$. In our model only this wave determines results of mental measurements. Thus (in our model) it seems to be impossible to find one to one correspondence between mental behaviour and neural activity. The flow of consciousness does not uniquely determine neural dynamics in brain.

10. Discussion

10.1. Why quantum-like formalism? One of the main reasons to expect that mental observables (including mental self-observables) should be described by the quantum-like (Hilbert space probabilistic) formalism is very high sensibility of neural structures to changes of contexts of measurement. Such a sensibility implies the violation of rules of classical probabilistic calculus and induces so called quantum probabilistic calculus, see [33], [35] for the detailed analysis. The main distinguishing feature of this quantum probabilistic calculus is intereference of probabilities of alternatives. Therefore a quantum-like structure of mental observables should imply interference effects for such observables. In [65] it was proposed the general scheme of mental measurements that could be used to find the interference effect. It may be that the corresponding statistical data was already collected somewhere. We need only to extract the intereefernce effect.

Another reason for quantum-like considerations is discrete structure of information processing in brain. It is natural to descrinbe this exchange by quanta of information by a quantum-like formalism. In particular, in our model we automatically obtained that basic mental observables such as e. g. mental energy have discrete spectra. We underline that *philosophy of organism* by *Alfred Whitehead* was one of the first philosophic doctrines in that fundamental proto-mental elements of reality, namely *actual occations,* had quantum (in the sense of discreteness) structure. The philosophy of organism was based on one-substance cosmology, see [1], p. 26:

> "*Descartes* and *Locke* maintained a two-substance ontology — *Descartes* explicitly, *Locke* by implication. *Descartes*, the mathematical physicists, emphasized his account to of corporeal substance; and *Locke*, the physician and the sociologist, confined himself to an account of mental substance. The philosophy of organism, in its scheme for one type of actual entities, adopts the view that *Locke's* account of mental substance embodies, in a very special form, a more penetrating philosophic description than *Descartes'* account of corporeal substance."

10.2. Quantum and conscious. In our model of consciousness as the process of (quantum-like) self-measurements over hierarchic neural structures the quantum structure plays important, but not determining role. There are many sensitive physical systems (not only microscopic, but also macroscopic) that could exhibit quantum-like behaviour, see [33]–[35] for the details. Thus to be quantum-like is not the sufficient condition to be conscious. There must be something else that is crucial in inducing consciousness. This consciousness determining factor may be quantum as well as classical (or a very special combination of classical and quantum factors).

It seems that the crucial point might be the ability to "feel" ensemble probability distribution of information strings produced by neural activity. My conjecture is that such a feeling is the basis of mentality. In such a model a cognitive system reacts not to firings of individual neurons or even large populations of neurons, but to **integral probability distribution of firings.** If this is the really the case, then quantum-like probabilistic formalism would appear automatically, since this is the most general theory of transformations of context depending probabilities [35]–[39].

10.3. Why p-adic space? On the classical level the main distinguishing feature of our model is the ultrametric p-adic structure of the classical mental space. As we have already mentioned in section 3, ulrametricity is simply a topological reperesentation of hierarchy. Hence, the main classical feature of the model is its very special hierarchic structure. I think that the presence of such a hierarchic structure is the very important condition of cognition and consciousness. In principle, it is possible to consider general ultrametric cognitive models. I restrict myself to consideration of p-adic models, since there is the possibility to connect p-adic hierarchic model with frequency domain models.

However, the presence of the p-adic hierarchy is not sufficient to induce consciousness (nor even cognition). For example, spin glasses have hierarchic structures that in some cases could also be mathematically described by p-adic numbers, see [62], [63]. The crucial point may be a complex system of interconnections between the huge ensemble of hierarchic neural structures in brain.

10.4. Individual and ensemble interpretations. Large diversity of physical interpretations of the mathematical formalism of quantum mechanics is one of serious problems in quantum foundations. Different interpretations provide totally different views to physical reality (including the absence of such a reality at all), see e. g. [10]–[12], [15].

As a consequence of the great success of books by *R. Penrose* on quantum approach to mind, neurophysiologists, psychologists, cognitive scientists, and philosophers are now well familia with one very special interpretation, namely Penrose's *quantum gravity* improvement of the *conventional interpretation* of quantum mechanics.

The first question is: Why does the conventional interpretation need some improvements at all?

This was well explained in book [15] [8]. Conventional quantum theory is a hardly seek person who has been struggling during last 75 years with numerous

[8] Neurophysiologists, psychologists, cognitive scientists, and philosophers are lucky that *R. Penrose* does not support orthodox views to quantum theory. So in his books [14], [15] this theory was not presented in the rigid orthodox form.

mysteries and paradoxes, see e. g. [15], p. 237:

> ... yet it contains many mysteries ... it provides us with a very strange view of the world indeed.";

or *R. Feynman*:

> "It is all mysterious. And the more you look at it the more mysterious it seems."

Unfortunately, all these mysteries and paradoxes were automatically transmited to cognitive sciences. Some people enjoy this and they are happy to speak about mental nonlocality or mental collapse. It is the general attitude to couple the mystery of consciousness with some conventional quantum mysteries, e. g. the mystery of superposition. On the other hand, many realistically thinking neurophysiologists, psychologists, cognitive scietists, and philosophers dislike to use all such quantum tricky things as superposition of (e. g. position) states for an individual system, collapse, nonlocalty, death of reality in the cognitive framework. I strongly suupport this viewpoint.

There is no any possibility to go deeply into foundations of conventional quantum theory. I think that the crucial point is the *individual interpretation* of a wave function. The wave function is associated with an individual quantum system (in the orthodox approach — it gives the complete description). For example, the *individual interpretation* induces such a mysterious thing as superposition (e. g. position) states for an individual system. On the other hand, individual superposition immediately implies that

> "Quantum theory provides a superb description of physical reality on a small scale...", [15], p. 237.

As the superposition of states for individual macroscopic objects (e. g. cars) was never observed, conventional quantum theory should be applied on so called quantum scale. In particular, all cognitive models based on conventional quantum theory should go deeply beyond the macroscopic neural level, see [15], p.355:

> "It is hard to see how one could usefully consider a quantum superposition consisting of one neuron *firing,* and simultaneously *not firing.*"

Therefore all such models suffer of the huge gap between quantum micro and neural macro scales. Of course, there are various attempts to solve these problem. For example, in [15] there was proposed to use quantum coherence to produce some macro states by coherence of large ensembles of quantum systems.

Finally, we mention quantum gravity improvement of conventional quantum theory, [14], [15]. This is really an improvement and not cardinal change of conventional quantum ideology. It is an attempt to explain reduction as *"gravitationally induced state-vector reduction."* It would not be useful to discuss in biological journal the role of such an improvement of physical theory. However, for cognitive models, the use of quantum gravity arguments looks as just increasing of convetional quantum mystifcation. There is a new huge gap between quantum scale and Planck scale (10^{-33} cm). It is even less belivable that mind is induced by superpositions of mass states.

Quantum-like approach to cognitive modeling used in this paper is based on so called *ensemble interpretation* of quantum mechanics, see e. g. *L. Ballentine* [11]. By this interpretation (that was strongly supported by *A. Einstein*) a wave function is associated not with an individual physical system, but with a statistical ensemble of systems. The statistical approach has its advantiges and disadvantiges. In particular, there is no mystery of state reduction. Individual systems are not in superposition of different states. Superposition of wave functions is a purely statistical property of various ensembles of physical systems. One of the main problems of the statistical approach was the impossibility to get interference of probabilities on the basis of classical ensemble probability. Recently it was done in author's works [33]–[36] by taking into account context dependence of probabilities. The absence of the mysterious superposition for individual systems and operation with ensembles gives the possibility to apply the Hilbert space probabilsitic formalism, quantum-like theory, to ensembles of macroscopic systems. Well, we agree with *R. Penrose* that an individual neuron could not be in superposition of two states, but two ensembles of neurons (as well as the same ensemble at distinct moments) could demonstrate features of superposition.

10.5. Neural code and structure of mental space. Suppose that the coding system of a cognitive system is based on a frequency code. There exists an interval of physical time Δ such that a classical mental state (mental position) produced by a hierarchic chain of neurons is a sequence with coordinates given by numbers of oscillations for corresponding neurons during the interval Δ. This Δ depends on a cognitive system and even on a psychological function inside the same brain, namely $\Delta = \Delta_{\tau,F}$. Thus in our model the problem of the neural code is closely related to the problem of time-scaling in neural systems. For different Δ, we get different coding systems, and, consequently, different structures of mental spaces. The corresponding natural number p that determines the p-adic structure on the mental space is defined as the maximal number of oscillations that could be performed by neurons (in hierarchic chains of neurons working for some fixed psychological function) for the time interval Δ. The coding that is based on e. g. the 2-adic system induces the 2-adic mental space that differs crucially from the 5-adic (or 1997-adic) mental space induced by the 5-adic (or 1997-adic) system. As it was remarked in section 3, by changing the p-adic structure we change crucially dynamics. Hence, the right choice of the time scaling parameter Δ and corresponding $p = p_\Delta$ plays the important role in the creation of an adequate mathematical model for functioning of a psychological function.

10.6. Mental time. There might be some connection between the time scale parameter Δ of neural coding and *mental time.* There are strong experimental evidences, see e. g. *K. Mogi* [66], that a moment in mental time correlates with ≈ 100 ms of physical time for neural activity. In such a model the basic assumption is that the physical time required for the transmission of information over synapses is somehow neglected in the mental time. A moment in mental time is subserved by neural activities in different brain regions at different physical times.

10.7. Quantum-like models with p-adic valued functions. In series of works of the author and his collaborators, see, for example, [47], there was developed the formalism of quantum mechanics in that not only the classical configuration space, but also wave functions are p-adic. Originally this formalism was developed to serve to high energy physics, theory of p-adic strings. Later I used this formalism for cognitive modeling, see e. g. paper [29] on p-adic cognitive pilot wave model ("conscious field model") giving the very special realization of *Bohm-Hiley-Pylkkänen* ideas on active information. From the mathematical point of view the p-adic valued formalism looks more attractive that complex valued formalism developed by *Vladimirov* and *Volovich*, see e. g. [45]. In particular, here operators of mental position and motivation are well defined. However, there is a difficulty that induces strong prejudice against this p-adic valued formalism, namely the appearence of p-adic valued probablities. Despite very successful mathematical development of the theory with p-adic valued probabilities [47], it is clear that we cannot use it for ordinary measurements over physical and cognitive systems. In such measurements we always observe ordinary probabilities. Thus p-adic valued quantum-like formalism could not be used to describe traditional mental measurements over a cognitive system performed by external systems. As it was pointed out in [32], such p-adic probabilities (stabilization of frequencies in p-adic topology and chaotic behaviour of these frequencies in ordinary real topology) might appear in anomalous phenomena. In principle, such probabilities might be related to functioning of consciousness. It might be that consciousness uses self-measurements following to p-adic valued quantum-like theory. However, in the present paper we would not like to study such a model of consciousness.

10.8. Real and p-adic spaces. From the first sight in our model there is no direct connection between real continuous space that is traditionally used to describe classical states of material objects and p-adic hierarchic (treelike) spaces that it was proposed to use to decribe classical mental states of brain. From the first sight we follow to *Descaurtes* doctrine. Such an approach was strongly critisized from many sides. In particular, such a theory is not coherent, see *Whitehead* [1]. Of course, it would be nice to develop some classical and corresponding quantum-like models based on real/p-adic space. The real and p-adic parts of material–mental space would describe physical brain and "mental brain", respectively. First remark is that in general we could not work with the fixed p-adic structure. As we have already discussed, different cognitive systems and psychological functions can be based on different p-adic mental spaces. Thus in general model we have to use all p-adic spaces simultaneously. We remark that a mathematical topological structure unifying real and all p-adic numbers (for prime p) is well known. This is so called adelic space, see [45] on physical models over adels. The next natural step would be to apply adelic quantum-like formalism to measurements over material and cognitive systems. In adelic quantum-like model "the disastrous separation of body and mind, characteristic of philosophical systems which are in any important respect derived from Cartesianism" (see [1], p. 348) could be avoided, since adelic amplitudes would depend both on body (real) and mind (p-adic) variables.

10.9. Microtubules. Are neurons really basic elements for hierarchic mental coding? At the moment there are strongest neurophysiological evidences that this is really the case. Nevertheless, we could not totally reject other possibilities. In particular, last 20 years *S. Hameroff* and his collaborators, see e. g. [67], have been developing the model of consciousness based on quantum processes in microtubules. Hameroff's approach is a traditional quantum reductionst approach. Thus our paper has nothing to do with it. However, the general idea that microtubules play an important role in information processing in brain should be considered very seriously in quantum-like approach. Of course, in such a model the main role would be played by hierarchic organization of microtubules on classical level. Quantum-like formalism can be used to describe the corresponding mental amplitude.

10.10. Non-reductionism. The basic question of all quantum reductionst models of consciousness is

> "How is it that consciousness can arise from such seemingly unpromising ingredients as matter, space, and time? — [15], p. 419.

In our model consciousness has no direct relation to matter. It is a feature of very special hierarchic configuration of information described by the mental amplitude $\phi(x)$. By answering to *Penrose's* question

> "The physical phenomenon of consciousness?", [15], p. 406,

I say: *"Not! Consciousness is not a bio-physical phenomenon. It is a bio-information phenomenon."*

10.11. Quantative measure of consciousness. I was extremely facsinated by *Baars'* idea to consider consciousness as a variable [68]. The main problem is to find some numerical representation of such a consciousness-variable. In our model such a variable should be in some way connected with the basic probability distribution $P(t,x) = |\phi(t,x)|^2$. This is the probability that the concrete hierarchic configuration of firings (e. g. configuration of frequencies) is realized in brain at the moment t. Hence, if sufficiently many hierarchic chains of neurons produce x, then $P(t,x)$ is sufficiently large. The value of $P(t,x)$ by itself cannot be taken as a quantative measure of mentality.

For instance, suppose that $P(t,x) \equiv 1$ for all x. This is the uniform distribution on the p-adic space. We could not expect that such an amplitude with uniform activation of all classical mental states corresponds to a high level of mentality[9]. Conscious behaviour corresponds to a mixture of various motivations. Such a mixture is characterized by variation of the probability distribution $P(t,x)$. I propose the following numerical measure of consciousness (at mental state $\phi(t,x)$) :

$$\mathcal{M}_{\text{consciousness}} = \int_{\mathbf{Q}_p} \left[\left| D_x P(t,x) \right|^2 + \left| \frac{\partial P(t,x)}{\partial t} \right|^2 \right] dx$$

[9]In particular, the free mental wave $\phi_\xi(x)$ induces such a probability distribution. In such a state a cognitive system has the fixed motivation ξ. By proceeding with a fixed motivation (aim, task) a cognitive system τ performs not conscious, but merely AI-behaviour (for example, realization of a program given by the string of digits ξ).

10.12. Neural groups. The fundamental role that internally organized groups of neurons (and not individual neurons) play in processing of information in brain was discussed in details in *Edelman's* theory of neural groups selection (TNGS), [69]. Our model in that neural pathways are used as the neural (classical) basis for processing of information in brain is closely related to TNGS. Of course, we understood that our model may be oversimplified. It may that basic units should be not chains, but whole trees of neurons.

10.13. Does consciousness benefit from long neural pathways? Finally, we discuss one of the greatest mysteries of neuroanatomy, see, for example, [17], [52]–[54], [69], [15]. It seems that in the process of neural evolution cognitive systems tried to create for each psychological function as long neural pathways as possible. This mystery might be explained on the basis of our neural pathway coding model. If such a coding be really the case, then a cognitive system τ gets great benefits by extending neural pathways for some psychological function as long as possible. For example, let the neural code basis $p = 5$ and a psychological function F is based on very short pathways of the length $L = 2$. Then the corresponding mental space contains $N(5, 2) = 2^5 = 32$ points. Let now $p = 5$ and $L = 10000$. Then the corresponding mental space contains huge number of points $N(5, 10000) = 10^{20}$ points. On the latter (huge) mental space there can be realized mental amplitudes having essentially more complex behaviour (and, consequently, the measure of consciousness). It might be that this mental space extending argument can be used to explain spatial separation of various maps in brain, see e. g. *Edelman* [69].

Main results of this paper were presented in author's talk and poster at the Conferences "Toward a Science of Consciousness" in Shövde, Sweden (2001), and Arizona (2002), [70].

I would like to thank *S. Albeverio, L. Ballentine, E. Beltrametti, T. Hida, D. Greenberger, S. Gudder, I. Volovich, W. De Muynck, J. Summhammer, P. Lahti, J-A. Larsson, H. Atmanspacher, B. Coecke, S. Aerts, A. Peres, A. Holevo, E. Loubenets, L. Polley, A. Zeilinger, C. Fuchs, R. Gill, L. Hardy, S. Goldshtein, A. Plotnitsky, A. Shimony, R. Jozsa, J. Bub, C. Caves, K. Gustafsson, H. Bernstein* for fruitful (and rather critical) discussions on the structure of quantum formalism. I would like to thank *D. Amit, B. Hiley, S. Greenfield, P. Pilkkänen, G. Vitiello* for discussions on cognitive modeling.

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